Lecture 6

Universal Turing Machine

Oblivious Turing Machine

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add arbitrary number of 1s in the end that are ignored

Universal Turing Machine

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Theorem: There exists a TM U such that $\forall x, \alpha \in \{0,1\}^*$, $U(x, \alpha) = M_\alpha(x)$.

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output of M_α on x

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Theorem: There exists a TM U such that $\forall x, \alpha \in \{0,1\}^*$, $U(x, \alpha) = M_\alpha(x)$. Moreover, if M_α halts on x within T steps,

Universal Turing Machine

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Universal Turing Machine

Universal Turing Machine

U

Universal Turing Machine

U

Universal Turing Machine

Input tape

U

Universal Turing Machine

U {

Input tape

Work tape 1

Universal Turing Machine

U {

Input tape

Work tape 1

Work tape 2

Universal Turing Machine

U {
- Input tape
- Work tape 1
- Work tape 2
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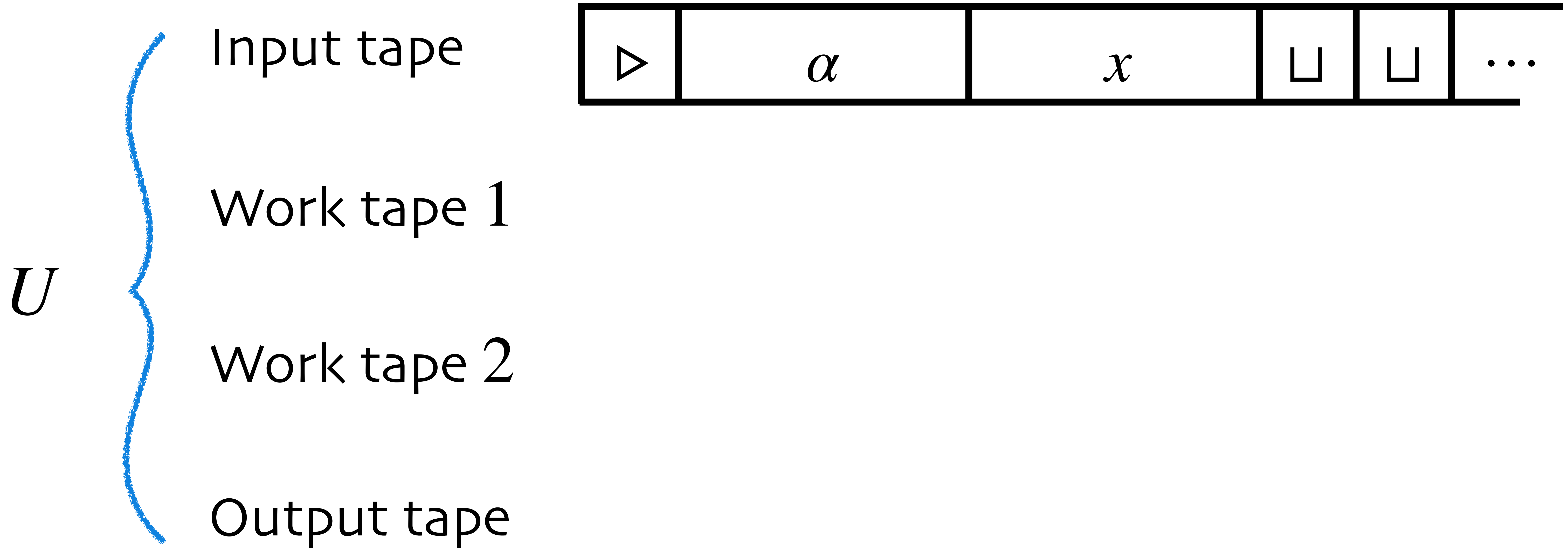
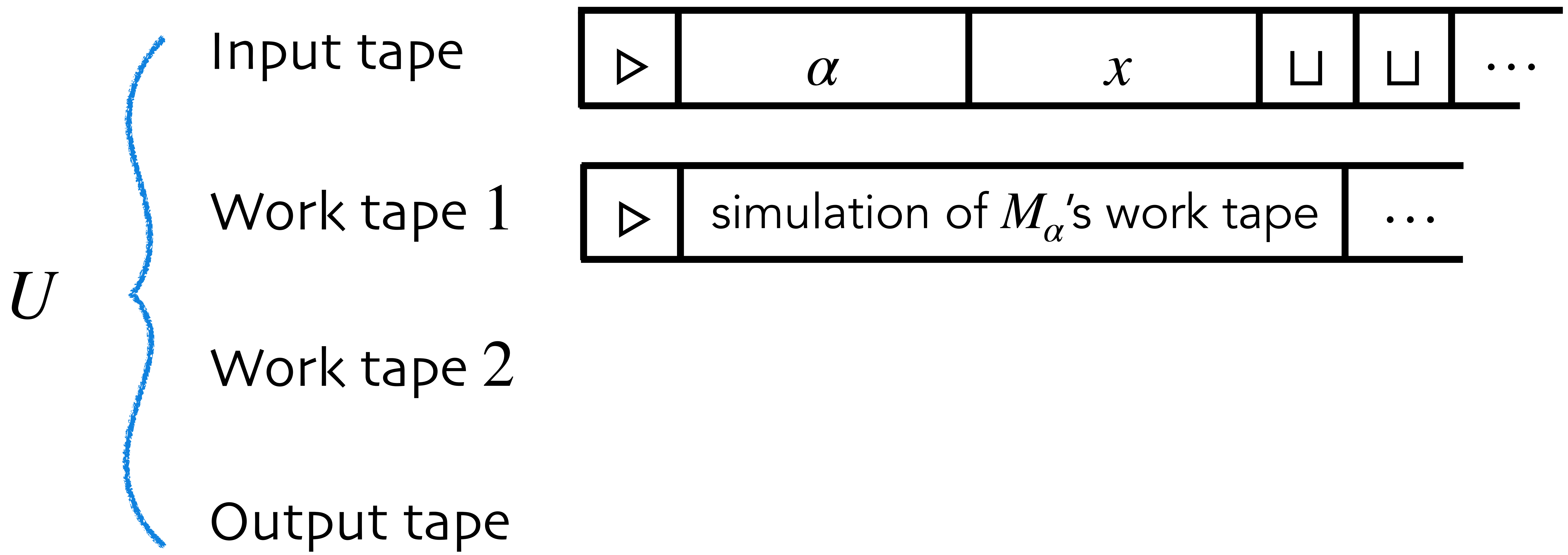
Universal Turing Machine



Universal Turing Machine



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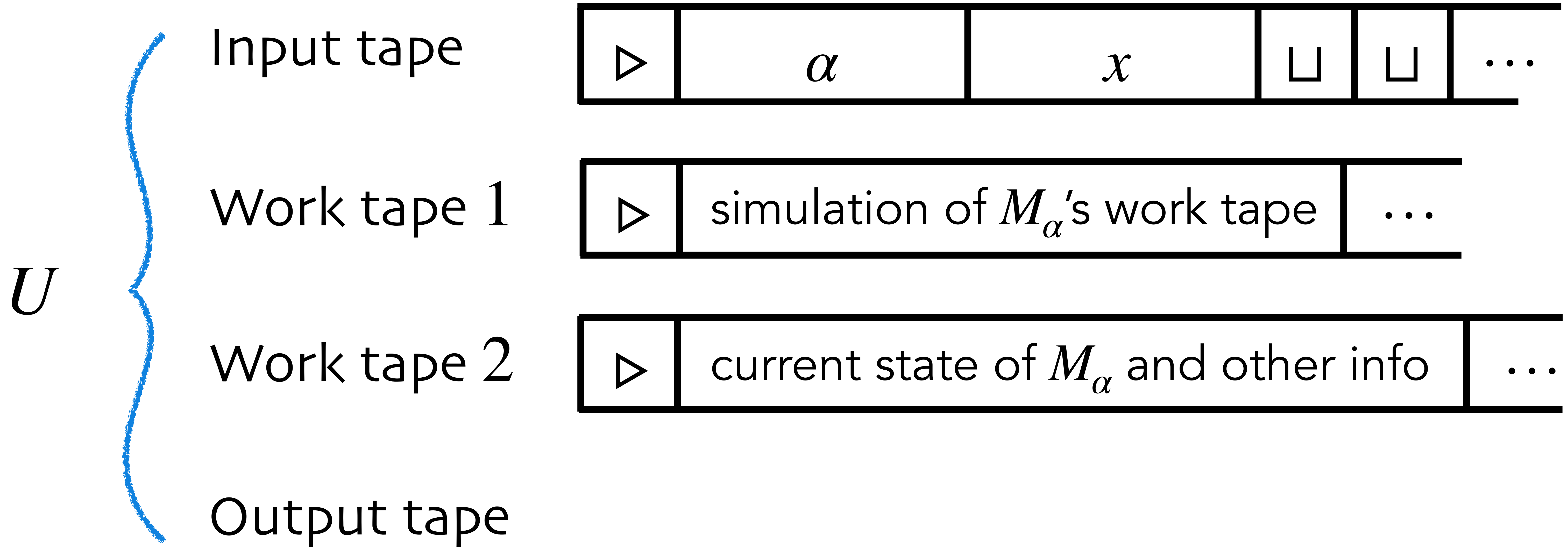

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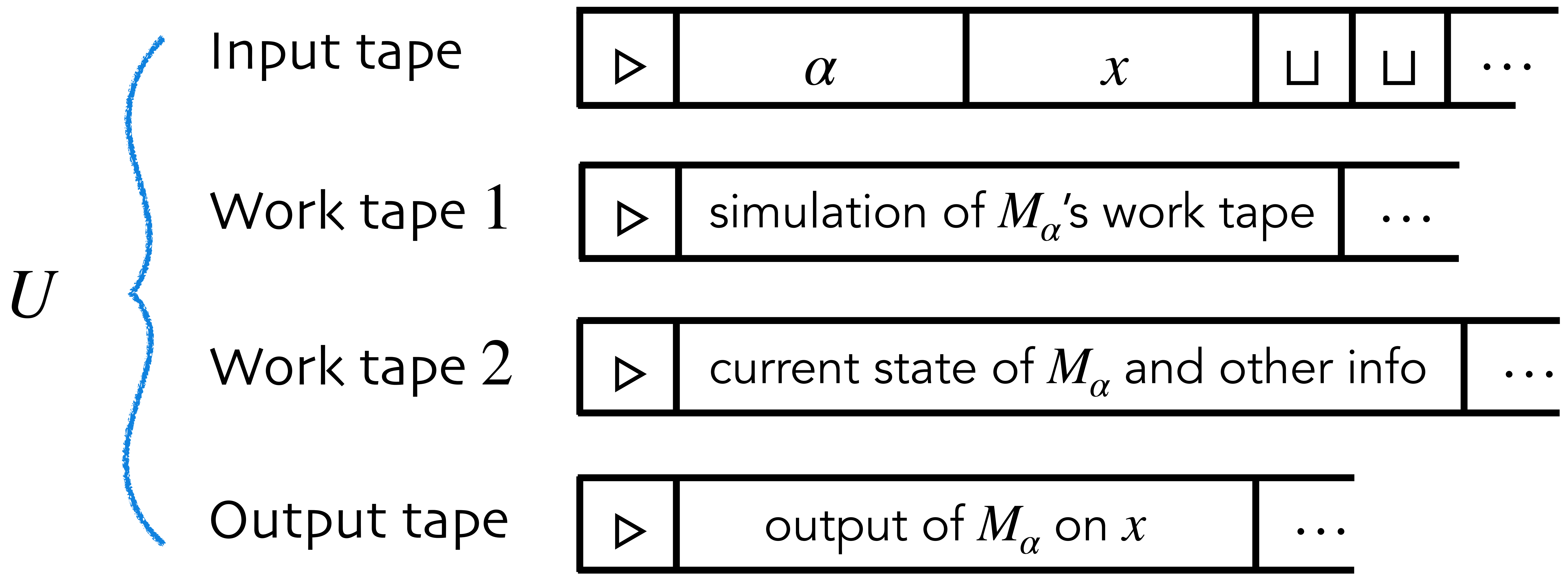

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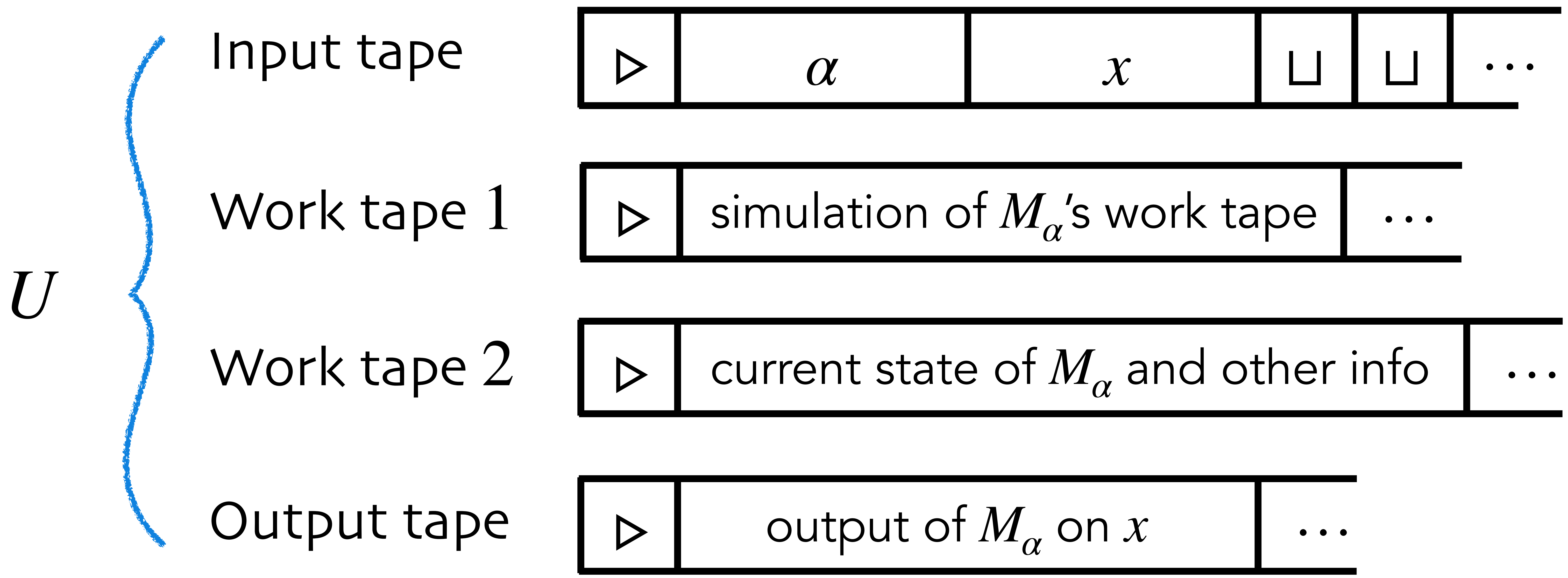
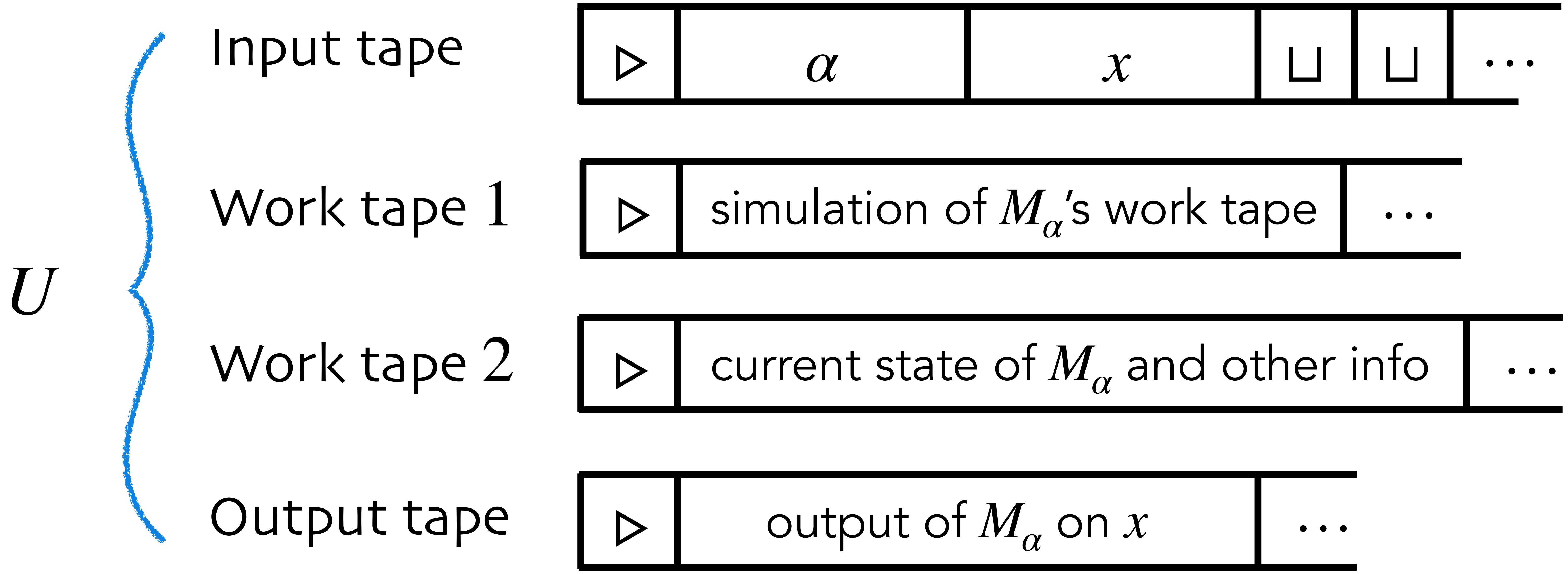


U's simulation of one step of M_α:

Universal Turing Machine



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- U reads the current symbols from *WT* 1 and writes them on *WT* 2.

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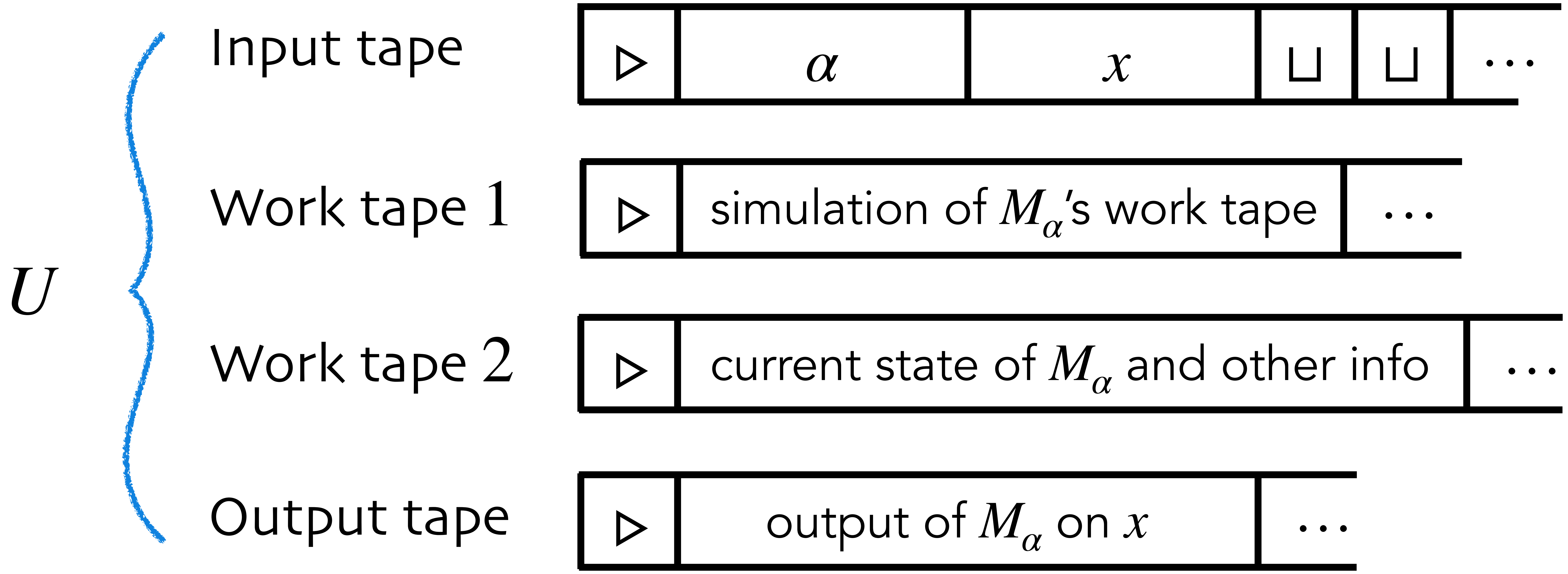

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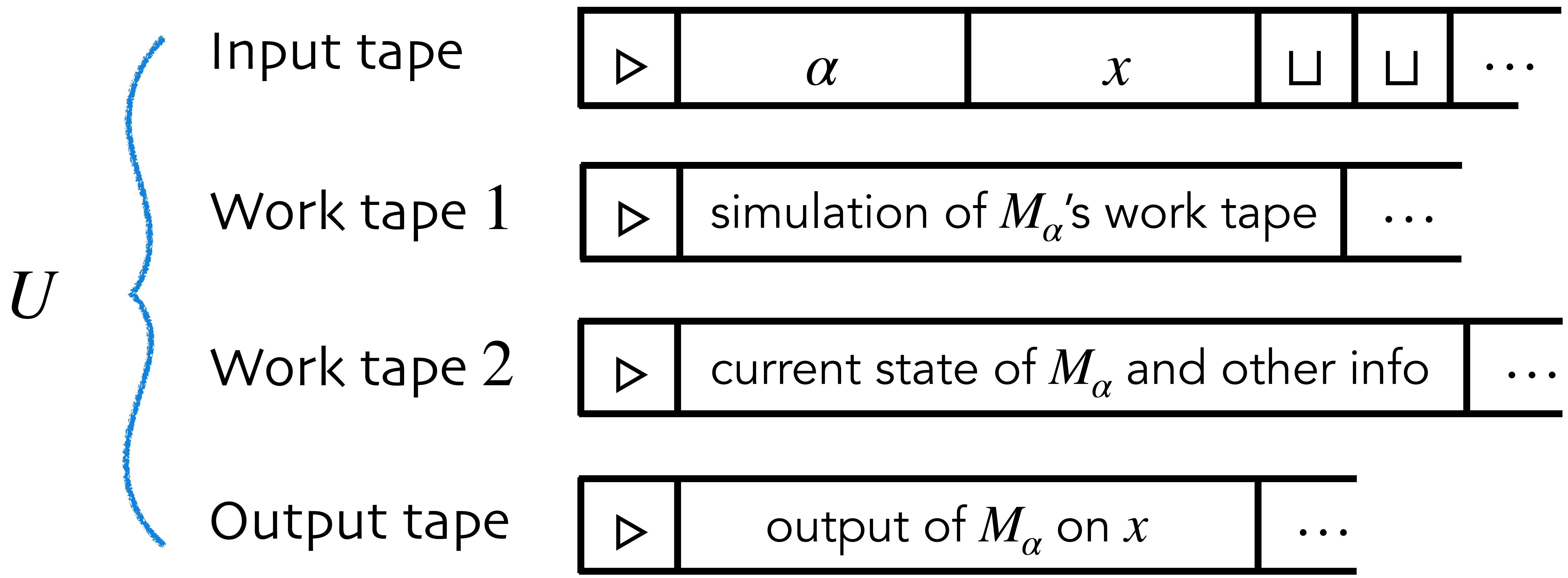
Universal Turing Machine



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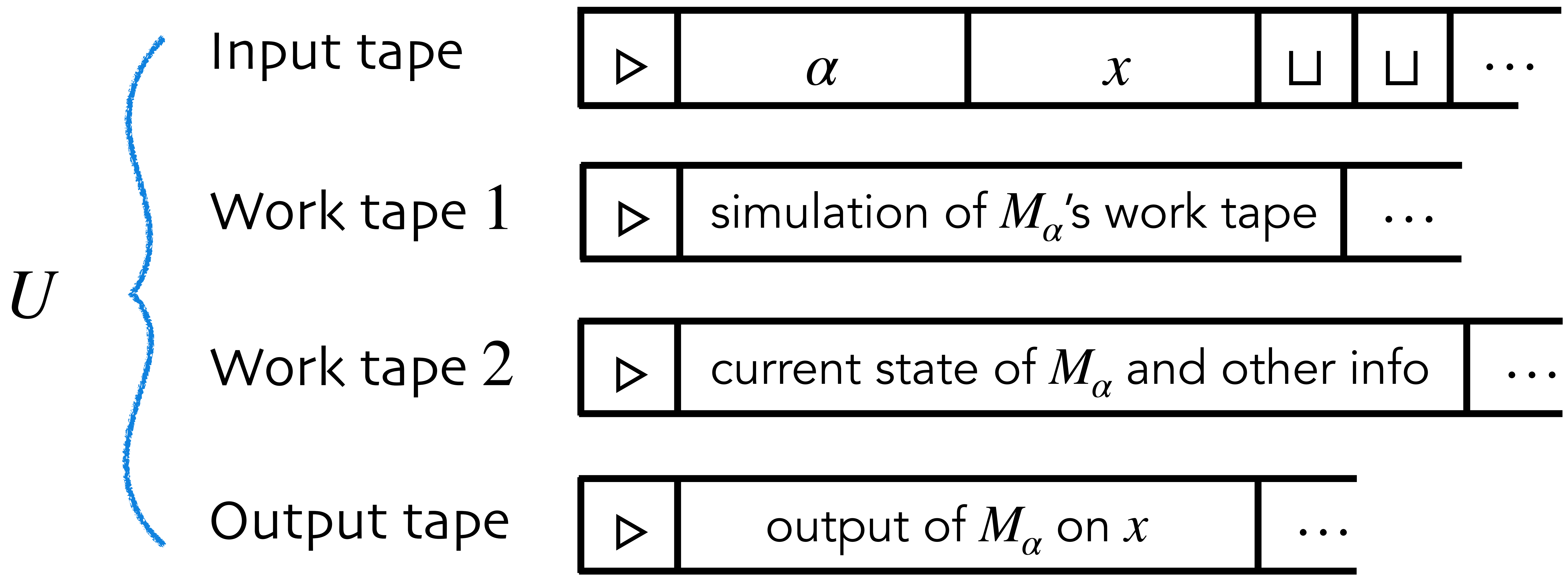
Universal Turing Machine



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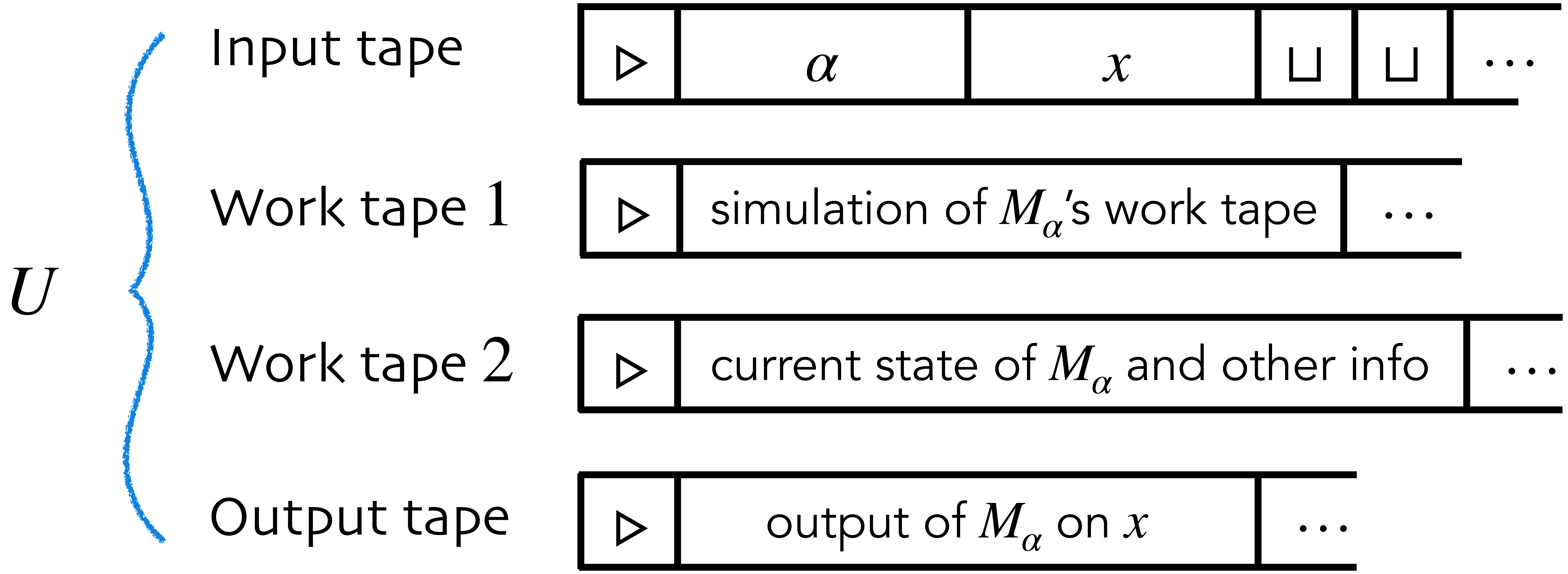
Universal Turing Machine



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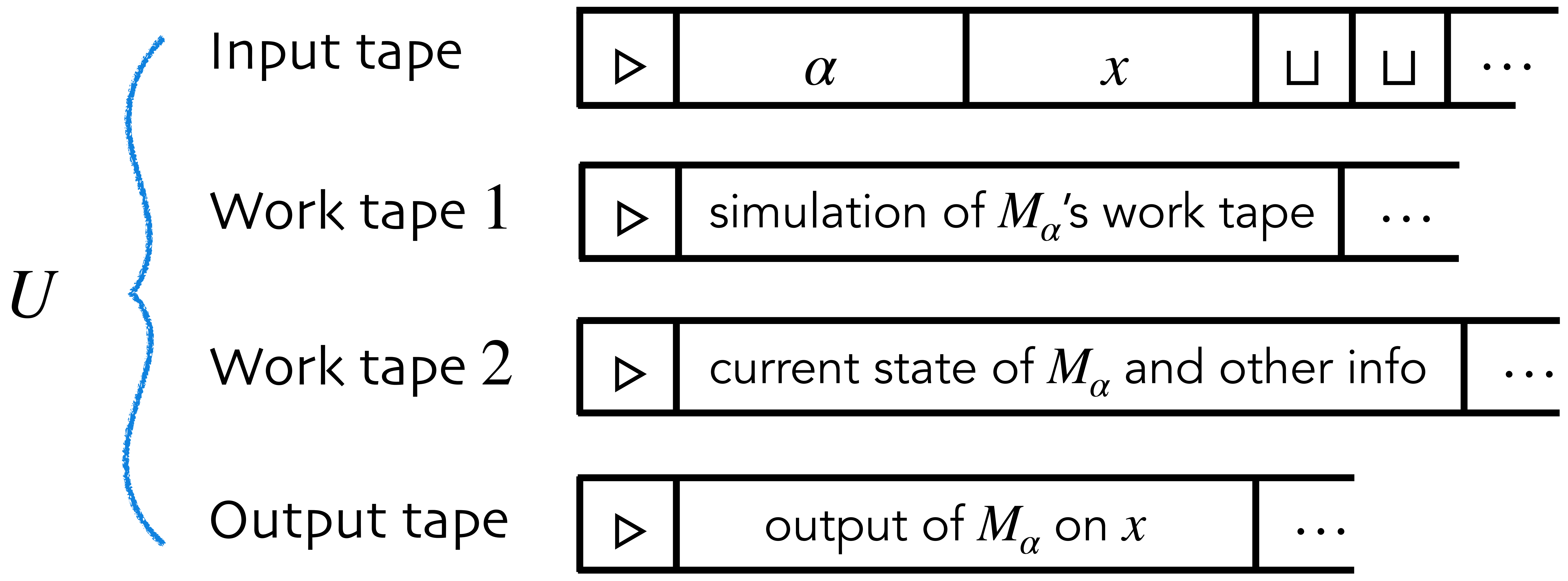

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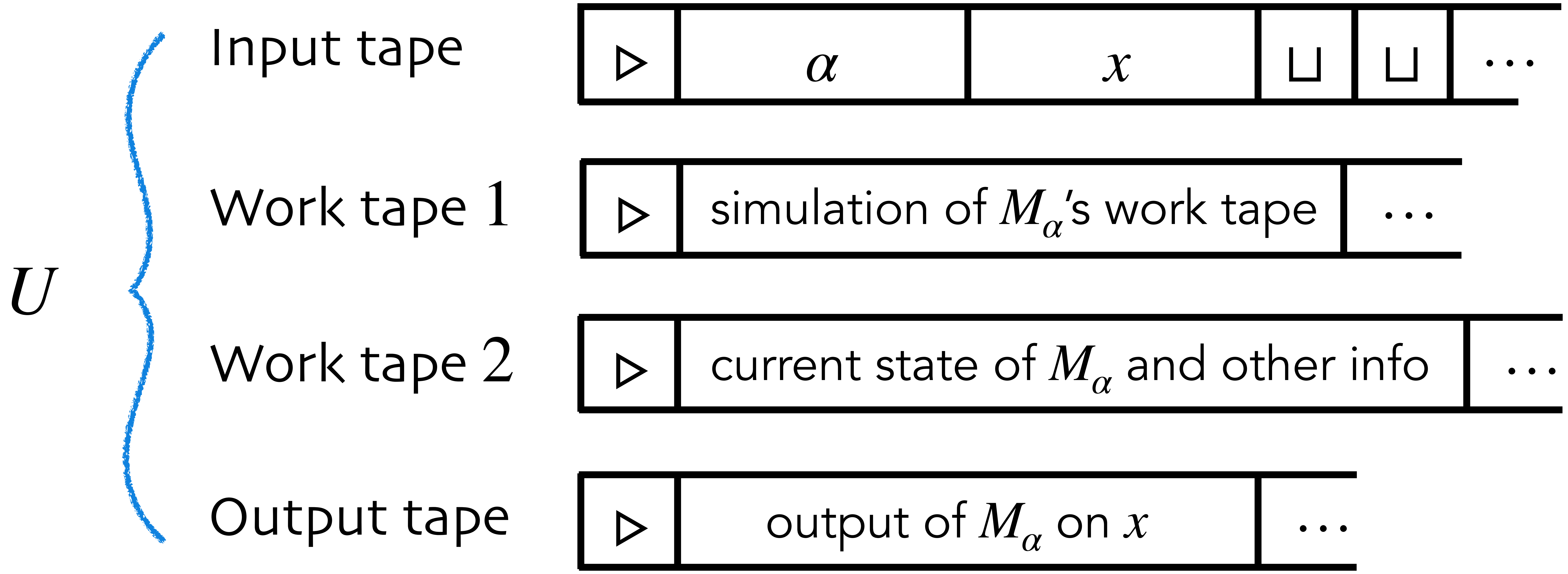

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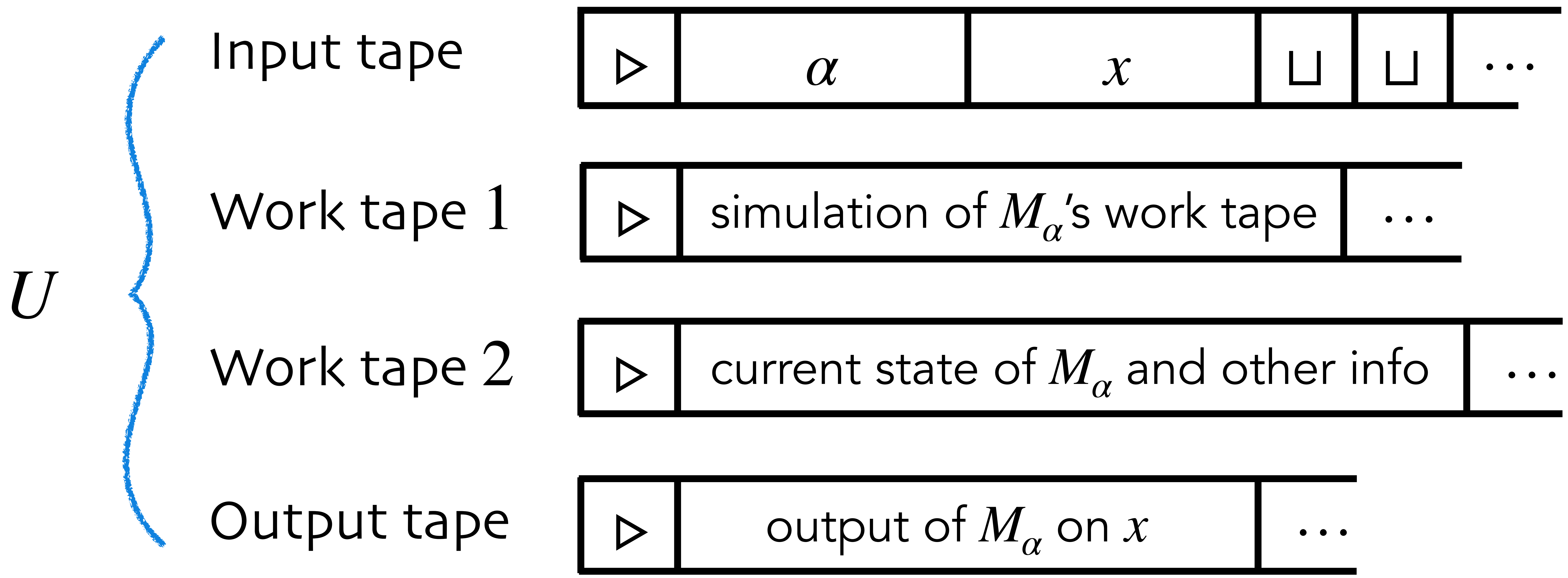

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